Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Tuesday 23 June 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/4C**

Further Mathematics

Advanced

Paper 4C: Further Mechanics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1/1/1/1/1/

1. Three particles of masses $3m$, $4m$ and $2m$ are placed at the points $(-2, 2)$, $(3, 1)$ and (p, p) respectively.

The value of p is such that the distance of the centre of mass of the three particles from the point $(0, 0)$ is as small as possible.

Find the value of p.

(7)

sum of moments = total mass × COM

Also written as

$$\sum_{i=1}^{n} m_i x_i = \bar{x} \sum_{i=1}^{n} m_i \quad \text{for } x \text{ axis}$$

$$\sum_{i=1}^{n} m_i y_i = \bar{y} \sum_{i=1}^{n} m_i \quad \text{for } y \text{ axis}$$

moments = force × perpendicular distance

for force we use mass × g but since 'g' is in every term it cancels out so we can just use mass

using the coordinates

Using vector form

$$3m\begin{pmatrix}-2\\2\end{pmatrix} + 4m\begin{pmatrix}3\\1\end{pmatrix} + 2m\begin{pmatrix}p\\p\end{pmatrix} = 9m\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}-6\\6\end{pmatrix} + \begin{pmatrix}12\\4\end{pmatrix} + \begin{pmatrix}2p\\2p\end{pmatrix} = \begin{pmatrix}9\bar{x}\\9\bar{y}\end{pmatrix}$$

$6 + 2p = 9\bar{x}$ $\qquad$ $10 + 2p = 9\bar{y}$

$\bar{x} = \dfrac{6+2p}{9}$ $\qquad$ $\bar{y} = \dfrac{10+2p}{9}$

Distance from $(0,0)$: $\sqrt{\left(\dfrac{6+2p}{9}\right)^2 + \left(\dfrac{10+2p}{9}\right)^2} = \sqrt{\dfrac{4p^2+24p+36}{81} + \dfrac{4p^2+40p+100}{81}}$

$$= \sqrt{\frac{8p^2+64p+136}{81}}$$

minimum distance from $(0,0)$ $\sqrt{\dfrac{8p^2+64p+136}{81}}$ } for this to be of minimum value $8p^2 + 64p + 136$ must be at a minimum

↳ completing the square

$8(p^2 + 8p + 17)$

$8[(p+4)^2 + 17 - 16]$

$p = -4$

⇒ ALTERNATIVE = Calculus

$\dfrac{dy}{dx} = 2p + 8$

at minimum, $2p + 8 = 0$

$2p = -8$

$p = -4$

Question 1 continued

(Total for Question 1 is 7 marks)

2.

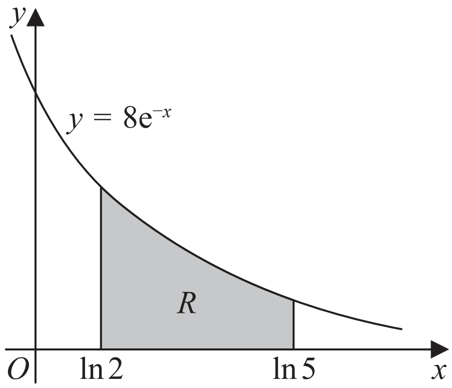


Figure 1

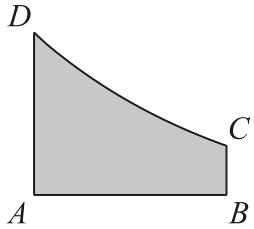


Figure 2

A uniform plane figure R, shown shaded in Figure 1, is bounded by the x-axis, the line with equation $x = \ln 5$, the curve with equation $y = 8e^{-x}$ and the line with equation $x = \ln 2$. The unit of length on each axis is one metre.

The area of R is $2.4\,\text{m}^2$

The centre of mass of R is at the point with coordinates $(\bar{x}, \bar{y})$.

(a) Use algebraic integration to show that $\bar{y} = 1.4$

(4)

Figure 2 shows a uniform lamina $ABCD$, which is the same size and shape as R. The lamina is freely suspended from C and hangs in equilibrium with CB at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ

(6)

a) $M\bar{y} = \int_a^b \frac{1}{2}\rho y^2\, dx$ To find COM of lamina under y between a and b in y axis

Applying formulae where ρ = mass per unit area (cancels out anyways)

$2.4\rho\bar{y} = \frac{1}{2}\rho\int y^2\, dx$ (cancelling out ρ)

$2.4\bar{y} = \frac{1}{2}\int_{\ln 2}^{\ln 5} 64e^{-2x}\, dx$

$2.4\bar{y} = \left[-16e^{-2x}\right]_{\ln 2}^{\ln 5}$

$2.4\bar{y} = -16e^{-2\ln 5} - \left(-16e^{-2\ln 2}\right)$

$2.4\bar{y} = -16\left(\frac{1}{25}\right) + 16\left(\frac{1}{4}\right)$

$2.4\bar{y} = \frac{84}{25}$

$\bar{y} = \frac{7}{5}$

$= 1.4$

Question 2 continued

b) $M\bar{x} = \int_a^b \rho xy \, dx$

To find COM of lamina under y between a and b in x axis where ρ = mass per unit area (cancels out anyways)

Applying formulae

$2.4\bar{x} = \int_{\ln 2}^{\ln 5} 8xe^{-x} \, dx$

Integration by parts

Let $u = 8x$ $\quad v = -e^{-x}$

$\frac{du}{dx} = 8$ $\quad \frac{dv}{dx} = e^{-x}$

$uv - \int v \frac{du}{dx}$

$= -8xe^{-x} - \int -8e^{-x} \, dx$

$= -8xe^{-x} - 8e^{-x} + c$

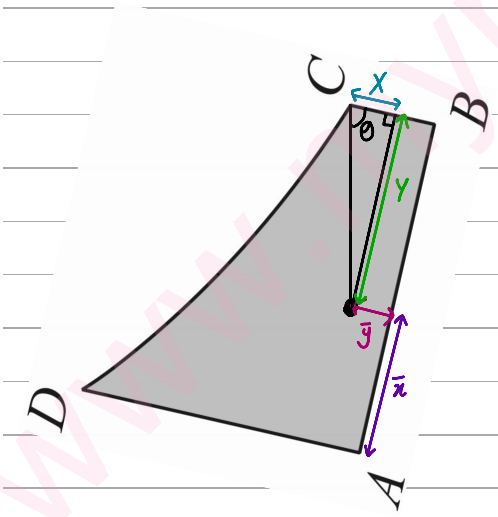
$2.4\bar{x} = \left[-8xe^{-x} - 8e^{-x}\right]_{\ln 2}^{\ln 5}$

$2.4\bar{x} = -8(\ln 5)e^{-\ln 5} - 8e^{-\ln 5} - \left(-8(\ln 2)e^{-\ln 2} - 8e^{-\ln 2}\right)$

$2.4\bar{x} = -\frac{8}{5}\ln 5 - \frac{8}{5} - (-4\ln 2 - 4)$

$2.4\bar{x} = -\frac{8}{5}(\ln 5 + 1) + 4(\ln 2 + 1)$

$\bar{x} = 1.08 \quad (3sf)$



At $x = \ln 5$, $y = 8e^{-\ln 5} = \frac{8}{5}$

$X = \frac{8}{5} - \bar{y}$

$X = \frac{8}{5} - \frac{7}{5}$

$X = \frac{1}{5}$

$Y = \ln 5 - \bar{x}$

use exact value from part b)

$\tan\theta = \frac{Y}{X} = \frac{\ln 5 - \bar{x}}{\frac{1}{5}} = 2.63\ldots$

$\theta = \arctan(2.63\ldots)$

$\theta = 69°$

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3. A particle P of mass 0.5 kg is moving along the positive x-axis in the direction of x increasing. At time t seconds $(t \geqslant 0)$, P is x metres from the origin O and the speed of P is $v\,\text{m s}^{-1}$. The resultant force acting on P is directed towards O and has magnitude kv^2 N, where k is a positive constant.

When $x = 1,\ v = 4$ and when $x = 2,\ v = 2$

(a) Show that $v = ab^x$, where a and b are constants to be found.

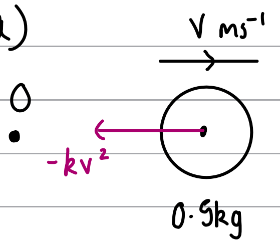
(6)

The time taken for the speed of P to decrease from $4\,\text{m s}^{-1}$ to $2\,\text{m s}^{-1}$ is T seconds.

(b) Show that $T = \dfrac{1}{4\ln 2}$

(4)

a)



$F = mv\dfrac{dv}{dx}$

Forming differential equation

$$0.5v\frac{dv}{dx} = -kv^2$$

Separating variables and integrating

$$\int \frac{1}{2v}\,dv = \int -k\,dx$$

$$\frac{1}{2}\ln v = -kx + C$$

Applying conditions

$x=1,\ v=4$: $\frac{1}{2}\ln 4 = -k + c$ ①

$x=2,\ v=2$: $\frac{1}{2}\ln 2 = -2k + c$ ②

① − ②: $\frac{1}{2}\ln 2 = k$ ③

③ into ①: $\frac{1}{2}\ln 4 = -\frac{1}{2}\ln 2 + c$

$c = \frac{1}{2}\ln 4 + \frac{1}{2}\ln 2$

$c = \frac{3}{2}\ln 2$

$$\therefore \frac{1}{2}\ln v = -\frac{1}{2}x\ln 2 + \frac{3}{2}\ln 2$$

$$\ln v = -x\ln 2 + 3\ln 2$$

$$\ln v = \ln\left(\left(\tfrac{1}{2}\right)^x\right) + \ln 8$$

$$\ln v = \ln\left(8\cdot\left(\tfrac{1}{2}\right)^x\right)$$ raise to $e^{\square}$

$$v = 8 \times \left(\tfrac{1}{2}\right)^x$$

where $a = 8$ and $b = \frac{1}{2}$

Question 3 continued

b) $F = m\frac{dv}{dt}$

Applying formulae

$0.5\frac{dv}{dt} = -kv^2$

Substitute k found in a)

$\frac{dv}{dt} = -\ln 2 \; v^2$

$\int_4^2 \frac{1}{v^2}\, dv = \int_0^T -\ln 2 \; dt$

Separating variables + integrating with boundaries 4 to 2 for v and 0 to T for t

$\left[-\frac{1}{v}\right]_4^2 = \left[-t\ln 2\right]_0^T$

$-\frac{1}{2} + \frac{1}{4} = -T\ln 2$

$T = \frac{1}{4\ln 2}$

Question 3 continued

P 6 2 6 8 4 R A 0 1 0 2 8

Question 3 continued

(Total for Question 3 is 10 marks)

4.

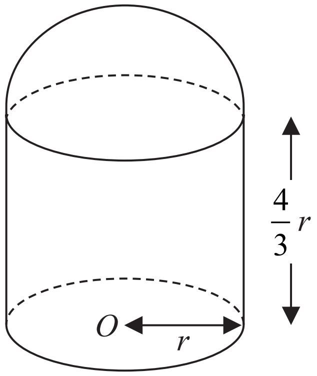


Figure 3

A uniform solid cylinder of base radius r and height $\frac{4}{3}r$ has the same density as a uniform solid hemisphere of radius r. The plane face of the hemisphere is joined to a plane face of the cylinder to form the composite solid S shown in Figure 3.
The point O is the centre of the plane face of S.

(a) Show that the distance from O to the centre of mass of S is $\frac{73}{72}r$

(4)

The solid S is placed with its plane face on a rough horizontal plane. The coefficient of friction between S and the plane is μ. A horizontal force P is applied to the highest point of S. The magnitude of P is gradually increased.

(b) Find the range of values of μ for which S will slide before it starts to tilt.

(5)

a)

Shape	Mass	Mass Ratio	COM from base
Hemisphere	$\frac{4}{3}\pi r^3 \times \frac{1}{2}\rho$	1	$\frac{4}{3}r + \frac{3}{8}r = \frac{41}{24}r$
Cylinder	$\frac{4}{3}\pi r^3 \rho$	2	$\frac{2}{3}r$
S	$2\pi r^3 \rho$	3	d

$\sum m_i x_i = \bar{x} \sum m_i$

where sum of moments of each component is equal to the singular moment through the COM

Use mass ratios to simplify calculation

Using sum of moments about diameter of the base

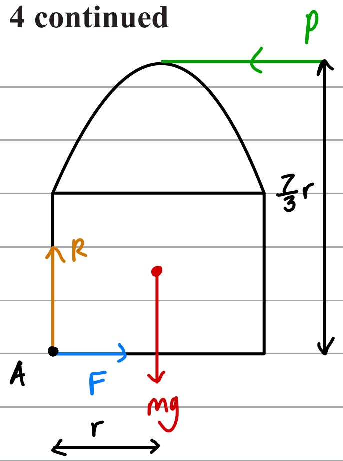
$1 \times \frac{41}{24}r + 2 \times \frac{2}{3}r = 3d$

$\frac{73}{24}r = 3d$

$d = \frac{73}{72}r$

Question 4 continued

b)



Resolving forces horizontally $(\rightarrow)$

$F = P$

where F = friction

$F_{max} = \mu R = \mu mg$

Solid slides if $P > \mu mg$

Taking moments about A

$\frac{7}{3} r P = r mg$

$P = \frac{3}{7} mg$ (if moments are balanced)

$\therefore$ Tilts if $P > \frac{3}{7} mg$ since solid will have a resultant moment

So slides before it starts to tilt when $\mu mg < \frac{3}{7} mg$

$\therefore \mu < \frac{3}{7}$

$\Rightarrow 0 < \mu < \frac{3}{7}$

since $\mu > 0$ for all μ anyways

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5.

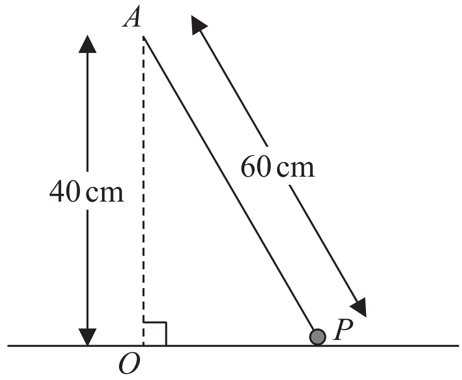


Figure 4

A particle P of mass 0.75 kg is attached to one end of a light inextensible string of length 60 cm. The other end of the string is attached to a fixed point A that is vertically above the point O on a smooth horizontal table, such that $OA = 40$ cm. The particle remains in contact with the table, with the string taut, and moves in a horizontal circle with centre O, as shown in Figure 4.

The particle is moving with a constant angular speed of 3 radians per second.

(a) Find (i) the tension in the string,

(ii) the normal reaction between P and the table.

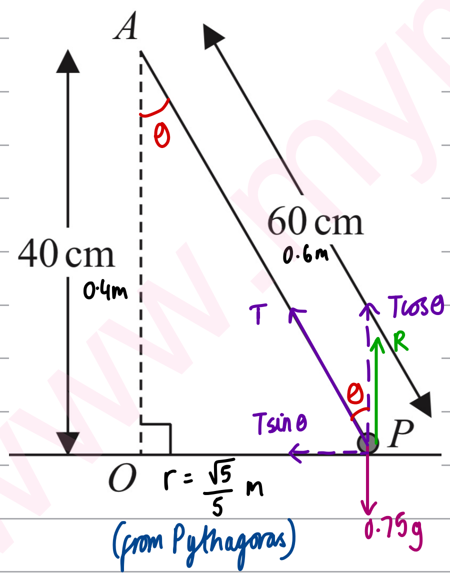
(7)

The angular speed of P is now gradually increased.

(b) Find the angular speed of P at the instant P loses contact with the table.

(4)

a)



Resolving forces vertically (↑):

$\cos\theta = \frac{40}{60} = \frac{2}{3}$

$0.75g = T\cos\theta + R$

$\sin\theta = \frac{\sqrt{5}}{5} \div 0.6 = \frac{\sqrt{5}}{3}$

$\frac{3g}{4} = \frac{2}{3}T + R$

Resolving forces horizontally (→): Circular motion

$T\sin\theta = m\omega^2 r$

$\frac{\sqrt{5}}{3}T = 0.75(3)^2\left(\frac{\sqrt{5}}{5}\right)$

$\frac{\sqrt{5}}{3}T = \frac{27\sqrt{5}}{20}$

i) $T = \frac{81}{20} = 4.05\text{N}$

ii) Substituting T into expression above

$R = \frac{3g}{4} - \frac{2}{3}(4.05) = 4.65\text{ N}$

P 6 2 6 8 4 R A 0 1 6 2 8

Question 5 continued

b) When ball loses contact with table: $R = 0$

$R(\uparrow)$: (vertically) $T\cos\theta = 0.75g$

$T = 0.75g \div \frac{2}{3}$

$T = \frac{441}{40}$ N

$T = 11.025$ N

$R(\leftarrow)$: (horizontally) $T\sin\theta = 0.75 \times \frac{\sqrt{5}}{5} w^2$

$w = \sqrt{\dfrac{T\sin\theta}{0.75 \times \frac{\sqrt{5}}{5}}}$

$= \sqrt{\dfrac{11.025\left(\frac{\sqrt{5}}{3}\right)}{0.75 \times \frac{\sqrt{5}}{5}}}$

$w = \frac{7\sqrt{2}}{2}$ rad s^{-1} ≈ 4.95 rad s^{-1}

Question 5 continued

P 6 2 6 8 4 R A 0 1 8 2 8

Question 5 continued

(Total for Question 5 is 11 marks)

P 6 2 6 8 4 R A 0 1 9 2 8

6.

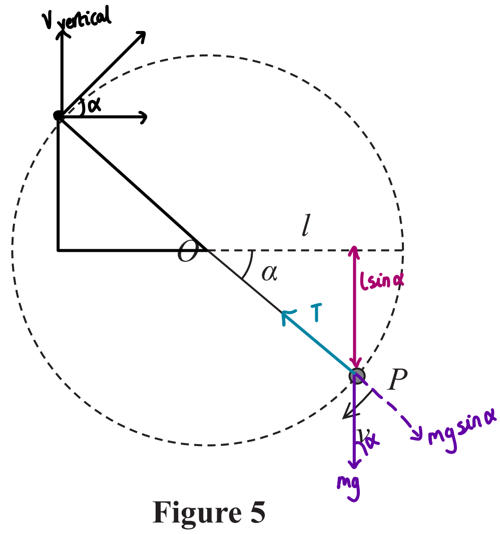


Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point O. The particle is held with the string taut and OP horizontal. The particle is then projected vertically downwards with speed u, where $u^2 = \frac{9}{5}gl$. When OP has turned through an angle α and the string is still taut, the speed of P is v, as shown in Figure 5. At this instant the tension in the string is T.

(a) Show that $T = 3mg\sin\alpha + \frac{9}{5}mg$

(6)

(b) Find, in terms of g and l, the speed of P at the instant when the string goes slack.

(3)

(c) Find, in terms of l, the greatest vertical height reached by P above the level of O.

(4)

a) Conservation of energy

$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgh$

$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgl\sin\alpha$

$v^2 = u^2 + 2gl\sin\alpha$

$v^2 = \frac{9gl}{5} + 2gl\sin\alpha$

Resolving forces towards centre of circle

$T - mg\sin\alpha = \frac{mv^2}{l}$

$T = mg\sin\alpha + \frac{mv^2}{l}$

$T = mg\sin\alpha + \frac{m\left(\frac{9gl}{5} + 2gl\sin\alpha\right)}{l}$

$T = mg\sin\alpha + \frac{9mg}{5} + 2mg\sin\alpha$

$T = 3mg\sin\alpha + \frac{9mg}{5}$

Question 6 continued

b) When string goes slack $T=0$

Set $T=0$

$$0 = 3mg\sin\alpha + \frac{9mg}{5}$$

$$-\frac{9mg}{5} = 3mg\sin\alpha$$

$$\sin\alpha = -\frac{3}{5}$$

Substituting into equation for v^2

$$v^2 = \frac{9gl}{5} + 2gl\sin\alpha$$

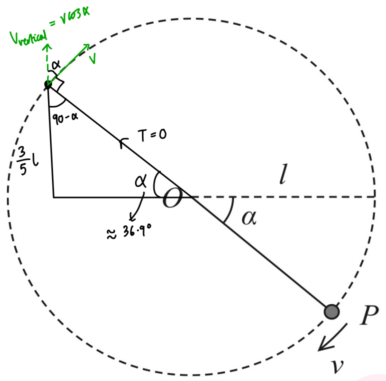
$$v = \sqrt{\frac{9gl}{5} + 2gl\left(-\frac{3}{5}\right)}$$

$$v = \sqrt{\frac{9gl}{5} - \frac{6gl}{5}}$$

$$v = \sqrt{\frac{3gl}{5}}$$

c) Greatest vertical height reached $\Rightarrow$ $T=0$ since string goes slack

initial vertical component of speed after string goes slack $= v\cos\alpha = \frac{4}{5}\sqrt{\frac{3gl}{5}}$



$\sin\alpha = -\frac{3}{5}$

$\alpha = -36.9°, 216.9°$

Then use SUVAT to calculate max height reached

$$v^2 = u^2 + 2as$$

$s = ?$

$u = \frac{4}{5}\sqrt{\frac{3gl}{5}}$

$v = 0$

$a = -g$

$$0 = \left(\frac{4}{5}\left(\sqrt{\frac{3gl}{5}}\right)\right)^2 + 2(-g)h$$

$$0 = \frac{16}{25} \times \frac{3gl}{5} - 2gh$$

$$h = \frac{\frac{16}{25} \times \frac{3gl}{5}}{2g} = \frac{24l}{125}$$

$\therefore$ Total height above O $= \frac{3l}{5} + \frac{24l}{125} = \frac{99l}{125}$

Question 6 continued

Question 6 continued

(Total for Question 6 is 13 marks)

7. A light elastic spring has natural length l and modulus of elasticity $4mg$. A particle P of mass m is attached to one end of the spring. The other end of the spring is attached to a fixed point A. The point B is vertically below A with $AB = \frac{7}{4}l$. The particle P is released from rest at B.

(a) Show that P moves with simple harmonic motion with period $\pi\sqrt{\frac{l}{g}}$
(7)

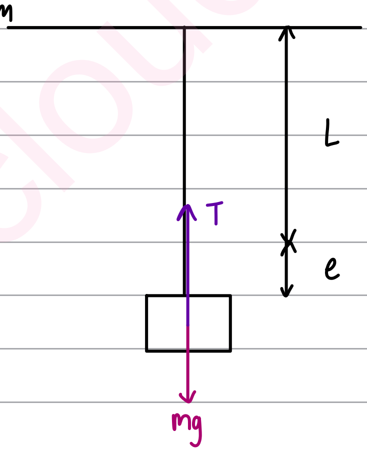
(b) Find, in terms of m, l and g, the maximum kinetic energy of P during the motion.
(3)

(c) Find the time within each complete oscillation for which the length of the spring is less than l.
(5)

a) $T = \frac{\lambda x}{L}$ where x = extension, λ = modulus of elasticity, L = natural length

$E = \frac{\lambda x^2}{2L}$ where E = elastic energy

At equilibrium



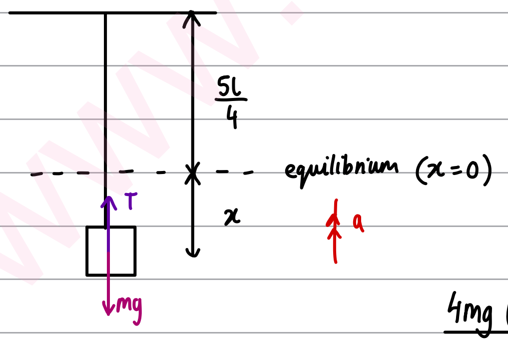
Resolving forces vertically at equilibrium

$T - mg = ma$ ($a = 0$)

$T = mg$

$\frac{4mg\,e}{L} = mg$

$e = \frac{1}{4}L$



Resolving forces vertically

$T - mg = -m\ddot{x}$

negative since acceleration opposes direction of displacement

$\frac{4mg(x+e)}{L} - mg = -m\ddot{x}$

$\frac{4mgx}{L} + \frac{4mge}{L} - mg = -m\ddot{x}$ ($\frac{4mge}{L} - mg = 0$)

$\ddot{x} = -\frac{4g}{L}x$

$\therefore$ SHM with $\omega = \sqrt{\frac{4g}{L}}$

Question 7 continued

b) $V_{max} = a\omega$ $\quad$ a = amplitude $\quad$ ω = angular speed

$KE_{max} = \frac{1}{2} m V_{max}^2$

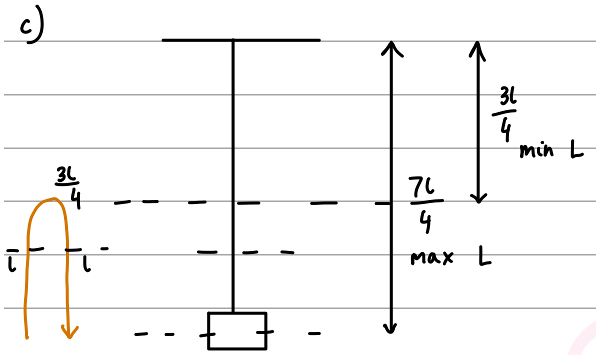
$V_{max} = \frac{l}{2}\sqrt{\frac{4g}{l}}$

$KE_{max} = \frac{1}{2}m\left(\frac{l}{2}\sqrt{\frac{4g}{l}}\right)^2$

$= \frac{1}{2}m \frac{l^2}{4}\left(\frac{4g}{l}\right)$

$KE_{max} = \frac{1}{2}mgl$

c)



↳ 1 complete oscillation

$x = a\cos\omega t$ (when starting from max/min displacement)

$x = \frac{l}{2}\cos t\sqrt{\frac{4g}{l}}$

when $x = -\frac{l}{4}$ (spring's length = l)

↑ (displacement from equilibrium)

$-\frac{l}{4} = \frac{l}{2}\cos t\sqrt{\frac{4g}{l}}$

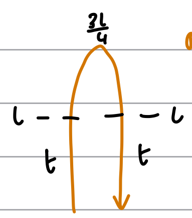
$\cos t\sqrt{\frac{4g}{l}} = -\frac{1}{2}$

$t\sqrt{\frac{4g}{l}} = \frac{2\pi}{3}$

$t = \frac{\pi}{3}\sqrt{\frac{l}{g}}$

Time period of one oscillation $= \frac{2\pi}{\omega} = 2\pi \div \sqrt{\frac{4g}{l}}$

$= \pi\sqrt{\frac{l}{g}}$



orange = T

∴ time where spring is less than l = $T - 2t$

$\pi\sqrt{\frac{l}{g}} - 2\left(\frac{\pi}{3}\sqrt{\frac{l}{g}}\right) = \frac{\pi}{3}\sqrt{\frac{l}{g}}$

Question 7 continued

P 6 2 6 8 4 R A 0 2 6 2 8

Question 7 continued

Turn over

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS